

UDK 681.51/.54

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## **OPTIMALSPEED CONTROLLER WITH PREDETERMINED DIFFICULTY**

**Introduction.** Today the engineering industry share of Ukraine is 12%, although in the early 90's it stood at 31% and was the basic industry of the country [1]. Dnipropetrovsk region engineering complex consists of 120 plants, which represent 10% of Ukrainian industry. The main improvement direction of economic development is the creation of the competitive engineering industry, that requires an infusion of new investment and the modernization of machine park.

Process automation in engineering and widespread adoption of CNC machines in industry led to the nomination of strict requirements for main motion machine tool electric drives. At the present stage the machine tool industry development is characterized by the transition to continuous speed variation and simplification of the electric drive kinematic structure, that allow to increase productivity and quality of metal through the rational choice of the cutting mode.

Frequent changes of the production program and the requirement increase of the product quality have led to the need of the rapid technology restructuring based on the demands of the market without substantial additional investment. To this effect the CNC machine tool use was introduced, which design is implemented with the principle of high speed metal cutting, also the modern highly dynamic electric drives are necessary to apply.

In modern CNC machines the control signal dynamic characteristics of the main motion drive determine to a considerable degree the operating efficiency of the machine tool. Instability speed requirements of main motion drives are the strictest compared to other drives. When the instrument penetrates into detail, the spindle speed reduction leads to a significant increase in the cutting force, which by turn – to further speed reduction. The speed instability of the main motion drive must not differ more than 5% from the set value in the whole range of speed regulation. The response rate requirements by changing the dynamic load torque are 0.1-0.3 s.

Because the modern machinery equipment of Ukraine by 80% is outdated and in need of modernization, the development of the machine tool electric drive that meets the requirements of quality and processing performance, is an actual task. Thus, there is a necessity to develop a deep adjustable main motion electric drive and control law that meets modern engineering requirements on the dynamic characteristics of the electric drive and the metal cutting mode features.

**Analysis of studies and publications.** At the moment the theory of optimal control allow to determine the structure and parameters of the regulators with any difficulty for linear, nonlinear and linearized control objects [2, 3]. However, optimization of the control law or the controller parameters and etc. is carried out often only by one performance criterion, such as motor heating [4], the control signal response accuracy [5] and the energy efficiency [6, 7]. The multicriteria optimization implementation of motor control systems will simultaneously improve technological and energy performance of engineering.

**Purpose.** The structure and parameter determination of the optimal control units for speed control of squirrel-cage induction motor, which enter into the admissible control device area.

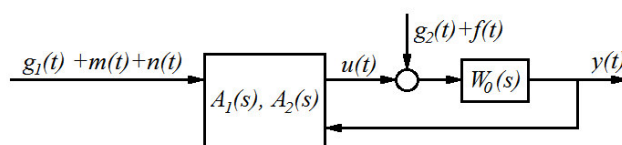
**Methodology.** For the synthesis of the optimal speed controller was used modified Hurwitz's criterion, that allows to obtain the parameters of the stabilization device. The synthesis problem is solved by minimization of the functionals, which reflects the requirements and limitations of the electromechanical system, that consider the practical implementation of the control system.

**Practical value.** The example of optimal correction link design with predetermined difficulty, that enter into the admissible control device area, which is limited by the obtained on the earlier stages of the study control units with maximum and minimum difficulty, is given. The defined using given method control unit group with different structures and parameters is the prerequisite for multicriteria analysis to obtain the optimal electromechanical system control units.

**Study materials.** The requirements for real electromechanical systems are not quite often only to ensure the performance quality by one criterion, such as the speed hold accuracy in the steady state, but by more criteria, such as the control signal response accuracy in dynamic mode, the uniform acceleration value without reference to the load torque, the implementation simplicity and etc., which are determined by the technological features of the control object. Therefore, to build a control system that simultaneously satisfies the criterion set, there is a need to optimize it for some objective functions that characterize different aspects of the object that is

the multicriteria design of the control system device. The problem solving of the best control device selecting starts with the formation of the possible controller set, from which those are selected that satisfy the imposed performance criteria and constraints. Then the Pareto-optimal set is allocated, from which the final structure and the parameters of the compensating devices are determined for further implementation and analysis. The acceptable control device set boundary is defined, on the one hand, by the control device configuration with maximum difficulty, and on the other hand—by the control device configuration with minimum difficulty. To choose the best control device the analysis of the all possible alternatives within certain boundary, the synthesis of the controllers with predetermined difficulty and the comparison of their performance indexes are carried out.

Consider the electromechanical system speed loop with speed feedback (Fig. 1). The control device for such a system has two inputs and one output. One of the system inputs is supplied by the reference signal  $g_1(t) + m(t) + n(t)$ , which consists of the variable by the certain law function  $g_1(t)$  and the random signal component  $m(t)$  with superimposed on it interfering signal  $n(t)$ . The other input is supplied by the measured control object output signal, which is subtracted from the reference signal. The output of the control device is the control signal  $u(t)$ , that together with the superimposed on it interfering signal, consisting of regular  $g_2(t)$  and a random  $f(t)$  components, is supplied to the control object input. The functions  $A_1(s), A_2(s)$  indicate the transfer functions of the control device relative to the reference signal and the output electromechanical system coordinate  $y(t)$ .



**Figure 1 – The block diagram of the closed-loop system**

Assume that the control object is a squirrel cage induction motor. From the classic representation of the induction motor mathematical model with the control by varying the stator supply voltage it can be seen that it contains the cross coupling by the stator current vector components. In case of the compensation or minimization of the cross coupling influence the stator voltage vector component variation can independently set the value of the rotor flux linkage and the motor speed. Then the flux linkage and speed control channels will be divided similar to a DC motor with separate excitation. Assume that the cross coupling by the stator current vector components is compensated and the inner current loop is optimized for the technical criterion, then the control object of the electromechanical system can be described as follows [ 8]

$$W_0(s) = \frac{1.5 \cdot p_{pp} \cdot k_2}{J \cdot k_c \cdot (T \cdot s + 1) \cdot s'}$$

where  $s$  – Laplace operator;  $p_{pp}$  – induction motor pole pairs;  $k_2 = L_m/L_2$  – dimensionless coefficient;  $L_m$  – magnetizing inductance, H;  $L_2$  – rotor inductance, H;  $J$  – motor inertia moment,  $kg \cdot m/s^2$ ;  $k_c$  – stator current sensor coefficient,  $V/A$ ;  $T$  – time constant of the stator loop, s.

The design of the control device with minimum difficulty begins by the device structure configuration choice, and then the transfer function parameters of the control device relative to the feedback  $A_2(s)$  are determined to ensure system stability. Within the selected control device configuration the compensating device construction alternatives are determined by the function  $A_2(s)$ . Thereafter the control device parameters are evaluated by the selected functional minimization.

To determine the transfer function parameters of the control device relative to feedback  $A_2(s)$  the modified Hurwitz's criterion is used. The closed-loop performance equation:

$$T(s) = P_0(s)G_2(s) + Q_0(s)V_2(s);$$

where

$$A_2(s) = \frac{V_2(s)}{G_2(s)}.$$

For the obtained expression work out a fractional rational function, which includes the polynomial with negative roots  $D(s)$ , its power is equal to the power of the performance polynomial  $T(s)$ , and the coefficients by the highest polynomial terms are equal to each other:

$$\Pi(s) = \frac{T(s)}{D(s)}.$$

The modified Hurwitz's criterion characterizes the vicinity of the polynomial  $T(s)$  and  $D(s)$  coefficients  $c_i$  and  $d_i$  respectively, according to which the mismatch minimum between them provides the stability of the system [3]:

$$I = \sum_{i=0}^n \rho_i |d_i - c_i|;$$

where  $\rho_i$  – weight coefficients.

The control device design is based on the transfer function configuration and structure relative to the feedback  $A_2(s)$ . The possible compensating link transfer functions are chosen so, that their structures match the structure of  $A_2(s)$ . One control device configuration and structure  $A_2(s)$  forms a set of possible compensating links. The compensating link transfer function parameters are determined by the minimization of the selected performance criteria similar to the used in the synthesis of the control device with the maximum difficulty. The transfer functions relative to the reference signal and superimposed noise can be written as [3]

$$\begin{aligned} \widehat{W}(s) &= \frac{A_1(s)W_0(s)}{1 + A_2(s)W_0(s)}; \\ \widetilde{H}(s) &= \frac{A_2(s)W_0(s)}{1 + A_2(s)W_0(s)}. \end{aligned}$$

Putting restrictions on the system astaticism assumes zero poles of the transfer functions relative to the direct control channel and the feedback channel. To ensure the  $a_2$ -th order of the system astaticism relatively to the noise  $g_2(t)$  the transfer function  $A_2(s)$  must be of the form

$$A_2(s) = \frac{V_2(s)}{G_2(s)} = \frac{V_2(s)}{s^{a_2}G_2^*(s)},$$

so the system astaticism is provided by any  $V_2(s)$  and  $G_2^*(s)$  if the polynomial  $V_2(s)$  has no zero roots. To ensure the  $a_1$ -th order of the astaticism relative to the reference signal the transfer function  $A_1(s)$  must be written as

$$A_1(s) = \frac{V_1(s)}{G_1(s)} = \frac{V_1(s)}{s^{a_1}G_1^*(s)},$$

so by choosing the parameters of polynomials  $G_1(s), G_2(s), V_1(s), V_2(s)$  the presence of  $a_1$  zero roots is provided in the polynomials [3]

$$C_1(s)G_2(s)G_1(s)P_0(s) + (V_2(s)G_1(s)C_1(s) - V_1(s)G_2(s)D_1(s))Q_0(s),$$

and the denominator roots must be negative.

At the earlier stages of research the compensating link transfer functions with the maximum and minimum difficulty relative to the reference and feedback signals were obtained:

$$W_{1max}(s) = \frac{1.389 \cdot 10^{-4} \cdot s^5 + 0.802 \cdot s^4 + 2.311 \cdot 10^3 \cdot s^3 + 3.385 \cdot 10^6 \cdot s^2 + 3.162 \cdot 10^8 \cdot s}{0.032 \cdot s^5 + 180.072 \cdot s^4 + 2.213 \cdot 10^4 \cdot s^3 + 4.996 \cdot 10^5 \cdot s^2 + 5.569 \cdot 10^6 \cdot s - 8.866 \cdot 10^{-9}};$$

$$W_{2max}(s) = \frac{0.49 \cdot s^3 + 732.133 \cdot s^2 + 1.826 \cdot 10^4 \cdot s + 2.276 \cdot 10^5}{1 \cdot 10^{-5} \cdot s^3 + 0.057 \cdot s^2 + 160.651 \cdot s + 2.276 \cdot 10^5};$$

$$W_{1min}(s) = \frac{16,0643s + 298.5951}{1.5329s};$$

$$W_{2min}(s) = 0,0318.$$

Then the controllers with predetermined difficulty should have the order from the first to the fifth and they exist and will meet necessary requirements. To determine the compensating link transfer functions the control device transfer function relative to the feedback signal should be written

$$A_2(s) = \frac{v_2s^2 + v_1s + v_0}{g_2s^2 + g_1s},$$

that satisfies the constraints on the first order astaticism relative to the noise  $g_2(t)$ , since the denominator of the transfer function has a zero root.

The closed-loop control system transfer function is

$$T(s) = v_2Ts^4 + (v_2 + v_1T)s^3 + (kg_2 + v_1 + v_0T)s^2 + (kg_1 + v_0)s + kg_0,$$

then the optimization function can be written as

$$I = \left| d_3 - \frac{v_2 + v_1T}{v_2T} \right| + \left| d_2 - \frac{kg_2 + v_1 + v_0T}{v_2T} \right| + \left| d_1 - \frac{kg_1 + v_0}{v_2T} \right| + \left| d_0 - \frac{kg_0}{v_2T} \right|,$$

where

$$k = \frac{1.5 \cdot p_{\text{нн}} \cdot k_2}{J \cdot k_c}.$$

The control device evaluation with predetermined difficulty was carried out for the squirrel cage induction motor 4A90L2U3. Taking into account the constraints on the coefficients of the optimization functional

$$d_0 > 0; d_1 > 0; d_2 > 0; d_3 > 0;$$

$$\begin{vmatrix} d_3 & 1 & 0 & 0 \\ d_1 & d_2 & d_3 & 1 \\ 0 & d_0 & d_1 & d_2 \\ 0 & 0 & 0 & d_0 \end{vmatrix} > 0,$$

that introduce constraints to obtain the performance equation roots only from the left side of the complex plane, obtain the control device transfer function relative to the feedback signal:

$$A_2(s) = \frac{316330s^2 + 11.53s - 44.02}{1422s^2 + 1.289s}.$$

After determining the structure and parameters of the function  $A_2(s)$ , which stabilizes the control object, proceed to the control device design based on its configuration. For the selected block diagram the relations between the control device transfer functions relative to the reference signal  $A_1(s)$  and feedback signal  $A_2(s)$  and the compensating links  $W_1(s)$ ,  $W_2(s)$  are as follows

$$A_1(s) = W_1(s);$$

$$A_2(s) = W_1(s)W_2(s).$$

From the first order astaticism condition relative to the reference signal the control device transfer function  $A_1(s)$  should have a zero pole, therefore the possible combinations of the compensating link transfer functions for the determined transfer function  $A_2(s)$  can be written as

$$W_1(s) = \frac{a_2s^2 + a_1s + a_0}{b_2s^2 + b_1s};$$

$$W_2(s) = c_0;$$

$$W_1(s) = \frac{a_1s + a_0}{b_1s}; W_2(s) = \frac{c_1s + c_0}{d_1s + d_0}.$$

The next step is to calculate the coefficients of the compensating link transfer functions based on the chosen criteria minimization. Consider the first link configuration alternative. Write the transfer functions of the electromechanical system relative to the reference signal and the noise

$$\widehat{W}(s) = \frac{A_1(s)W_0(s)}{1 + A_2(s)W_0(s)} = \frac{k(g_2s^2 + g_1s)(a_2s^2 + a_1s + a_0)}{(Ts + 1)(g_2s^2 + g_1s) + k(v_2s^2 + v_1s + v_0)};$$

$$\widetilde{H}(s) = \frac{A_2(s)W_0(s)}{1 + A_2(s)W_0(s)} = \frac{k(v_2s^2 + v_1s + v_0)}{(v_2s^2 + v_1s + v_0)(Ts + 1) + k(g_2s^2 + g_1s)}.$$

The desired transfer function of the regular reference signal component transform  $g_1(t)$  is assumed as  $U_1(s) = 1$ , and the desired transfer function of the regular noise signal component transform  $g_2(t)$  is assumed as  $U_2(s) = 0$ , and for the random components –  $P_1(s) = 0$  and  $P_2(s) = 0$ . As the control object contains no positive zeroes and poles, the components that impose the restrictions on the positive zero and pole compensation are not included to the functional. Then the functionals for the control device transfer function optimization problem solving relative to the reference signal take the following form

$$I_1 = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} \left| \left(1 - \widehat{W}(s)\right) \frac{1}{s} \right|^2 ds + \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} |s\widehat{W}(s)|^2 ds,$$

$$I_2 = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} \left| \left(1 - \widetilde{H}(s)\right) W_0(s) \right|^2 ds + \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} |s\widetilde{H}(s)|^2 ds.$$

After the optimization problem is solved the transfer function  $A_1(s)$  is obtained

$$A_1(s) = \frac{6793000s^2 + 362.1s - 1382}{1422s^2 + 1.289s}.$$

Based on the expressions of  $A_1(s)$  and  $A_2(s)$  the compensating link transfer functions for the direct control channel and the feedback channel can be written

$$W_1(s) = \frac{6793000s^2 + 362.1s - 1382}{1422s^2 + 1.289s};$$

$$W_2(s) = 0.0318.$$

For the second compensating link configuration alternative the transfer function synthesis is based on the statement, that the steady-state mode feedback transfer coefficient should ensure the compliance of the output speed value and the input reference signal value:

$$W_2(s) = \frac{c_1s + 10}{d_1s + 314,159},$$

where the control device transfer function relative to the reference signal can be found as follows

$$A_1(s) = \frac{(s - 0.0142)(s + 0.0143)(d_1s + 314.159)}{(s + 0.00091)(c_1s + 10)s}.$$

After the optimization problem is solved the compensating link transfer functions for the direct control channel and the feedback channel are

$$W_1(s) = \frac{3314s^3 + 67930s^2 + 2.947s - 13,82}{1.93s^3 + 14.24s^2 + 0.01289s};$$

$$W_2(s) = \frac{1,357s + 10}{15,319s + 314,159}.$$

For the calculated transfer functions were obtained decibel-log frequency response (Fig. 2) and the electromechanical system transient process graphs of the up to rated speed free acceleration and the rated load-on (Fig. 3), where the number of line corresponds to the configuration alternative.

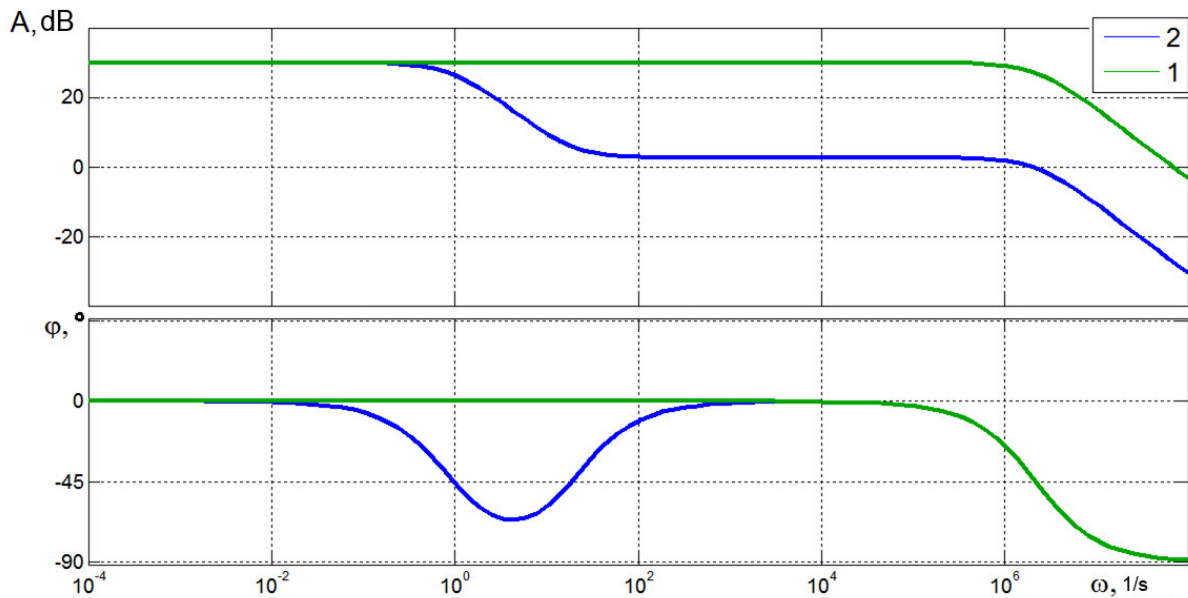


Figure 2 – Decibel-log frequency response of the developed control systems

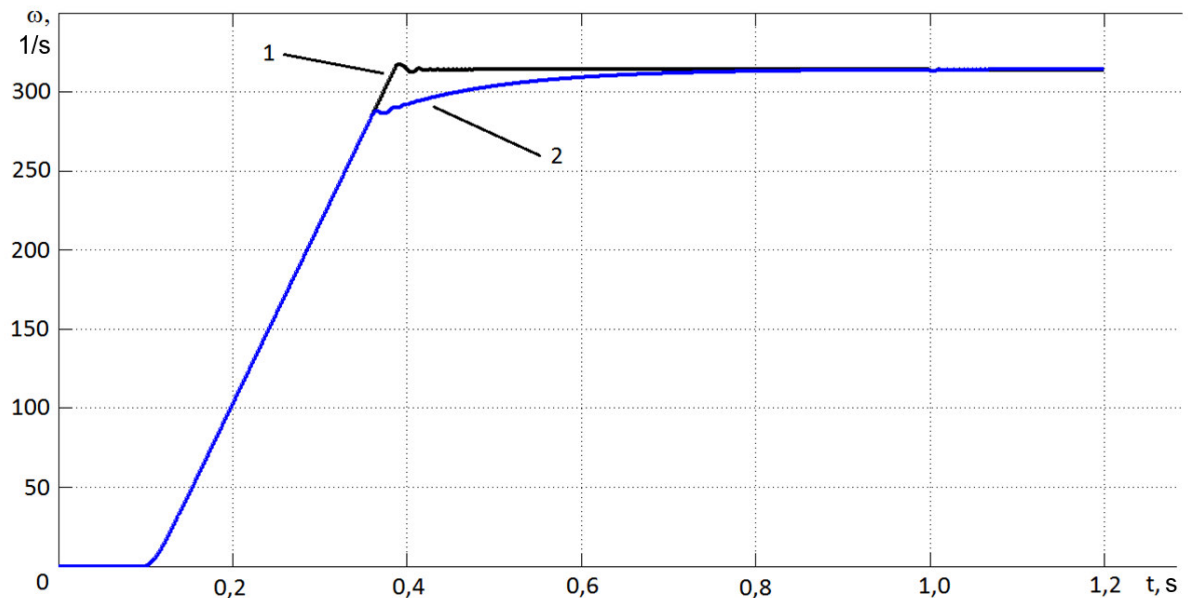


Figure 3 – Graphs of the electromechanical system transient processes

For the developed control devices semi-logarithmic integral sensitivity functions are calculated, that characterize the change of the entire system transfer function depending on the relative change of the component part transfer function [3]

$$D_{1i} = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} \left| \frac{d\ln\widehat{W}(s)}{d\ln W_i(s)} \widehat{W}(s) \right|^2 ds,$$

$$D_{2i} = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} \left| \frac{d\ln H(s)}{d\ln W_i(s)} H(s) \right|^2 ds, \quad i \geq 0,$$

where  $\widehat{W}(s)$  – the transfer function relative to the reference signal;  $H(s)$  – the transfer function relative to the noise  $g_2(t) + f(t)$ ;  $W_i(s)$  – the transfer function of the  $i$ -th object, that is a part of the control system.

The numerical function sensitivity evaluation through the fractional rational function coefficients indicates the presence of the integrand positive poles or poles that located on the imaginary axis, if the integral diverges, and is defined as [3]

$$I = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} K(s)K(-s)ds = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} |K(s)|^2 ds = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} \left| \frac{b_{n-1}s^{n-1} + \dots + b_1s + b_0}{a_n s^n + \dots + a_1s + a_0} \right|^2 ds =$$

$$= \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} \frac{c_{n-1}s^{2(n-1)} + c_{n-2}s^{2(n-2)} + \dots + c_1s^2 + c_0}{|a_n s^n + \dots + a_1s + a_0|^2} ds = \frac{(-1)^{n-1}N_n}{2a_n D_n},$$

where  $K(s)$  – the fractional rational function with negative poles;  $D_n$  – Hurwitz's determinant;  $N_n$  – the determinant, obtained from  $D_n$  by the substitution of the first column elements with the coefficients  $c_{n-1}, c_{n-2}, \dots, c_1, c_0$ .

The integral system sensitivity to the control object dynamic performance change function for the first compensating link configuration alternative:

$$D_1 = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} \left| \frac{d\ln\hat{W}(s)}{d\ln W_0(s)} \hat{W}(s) \right|^2 ds = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} |(1 - \tilde{H}(s))\hat{W}(s)|^2 ds = 6,805 \cdot 10^5,$$

and for the second configuration alternative:

$$D_2 = 3,688 \cdot 10^8.$$

From the semi-logarithmic sensitivity function values can be concluded that the first configuration alternative is less sensitive to the changes of the control object parameters, whereby it is easier to implement.

## Conclusions.

Obtained structures and optimal speed control device parameters for the induction motor within the acceptable set of optimal devices, which with the devices with maximum and minimum difficulty, are the initial data for the further multicriteria control system construction.

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*Рекомендовано до друку к-том техн. наук, доцентом Азюковським О.О.*